3D

5 – viewing and projection

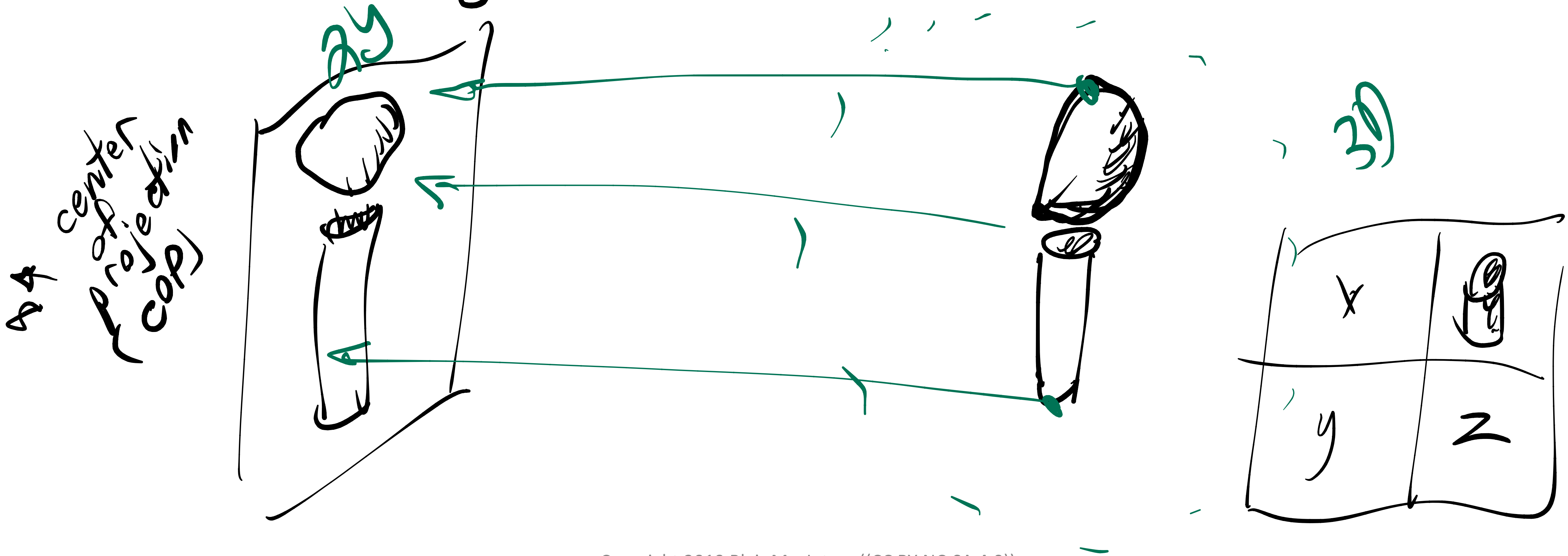
*

3D → 2D

Projection

moving points into a subspace

projecting 3D into 2D plane (view plane)



$$M = M_{T_1} M_{T_2} M_{T_3}$$

$$M^{-1} = M_{T_3}^{-1} M_{T_2}^{-1} M_{T_1}^{-1}$$

Viewing Transformations

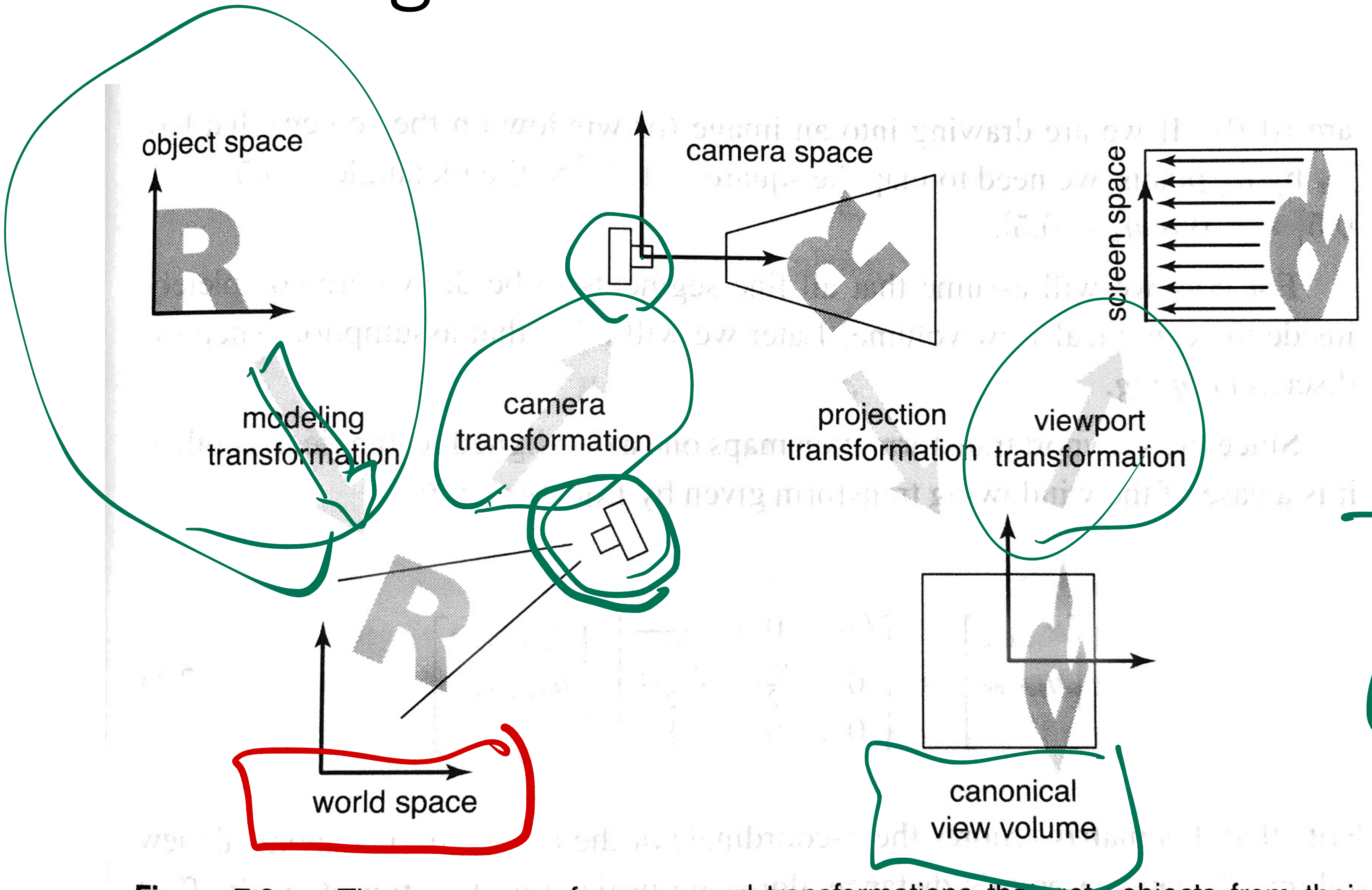
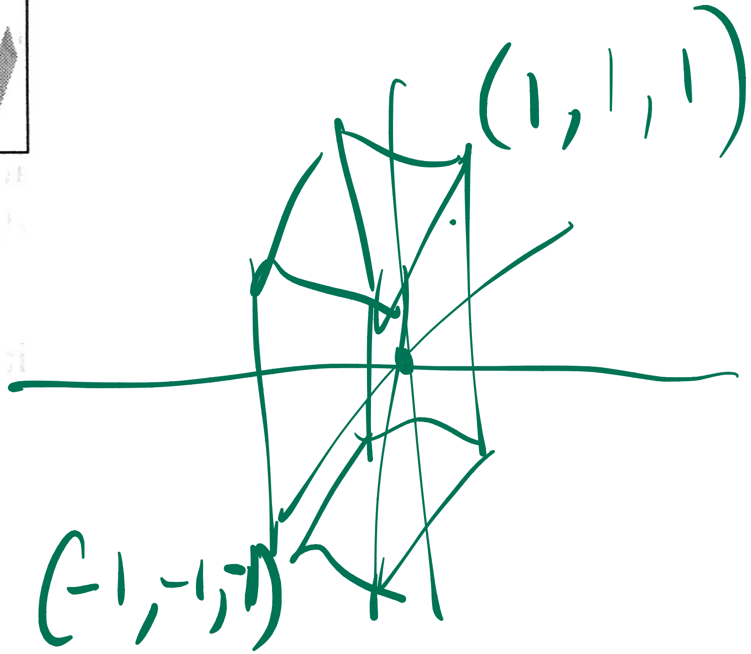


Figure 7.2. The sequence of spaces and transformations that gets objects from their original coordinates into screen space.



Goal: Matrices for everything

Want an M such that

$$M \quad = M_{vp} M_{projection} M_{cam}$$

One combined Matrix

Viewport

persp or ortho

Camera transform (world → camera)

Trans

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}$$

Camera Transforms

M_{cam} = transformation to camera pose (viewpoint and direction)

$$= R_{-z\,axis} \; \overrightarrow{T}_{origin}$$

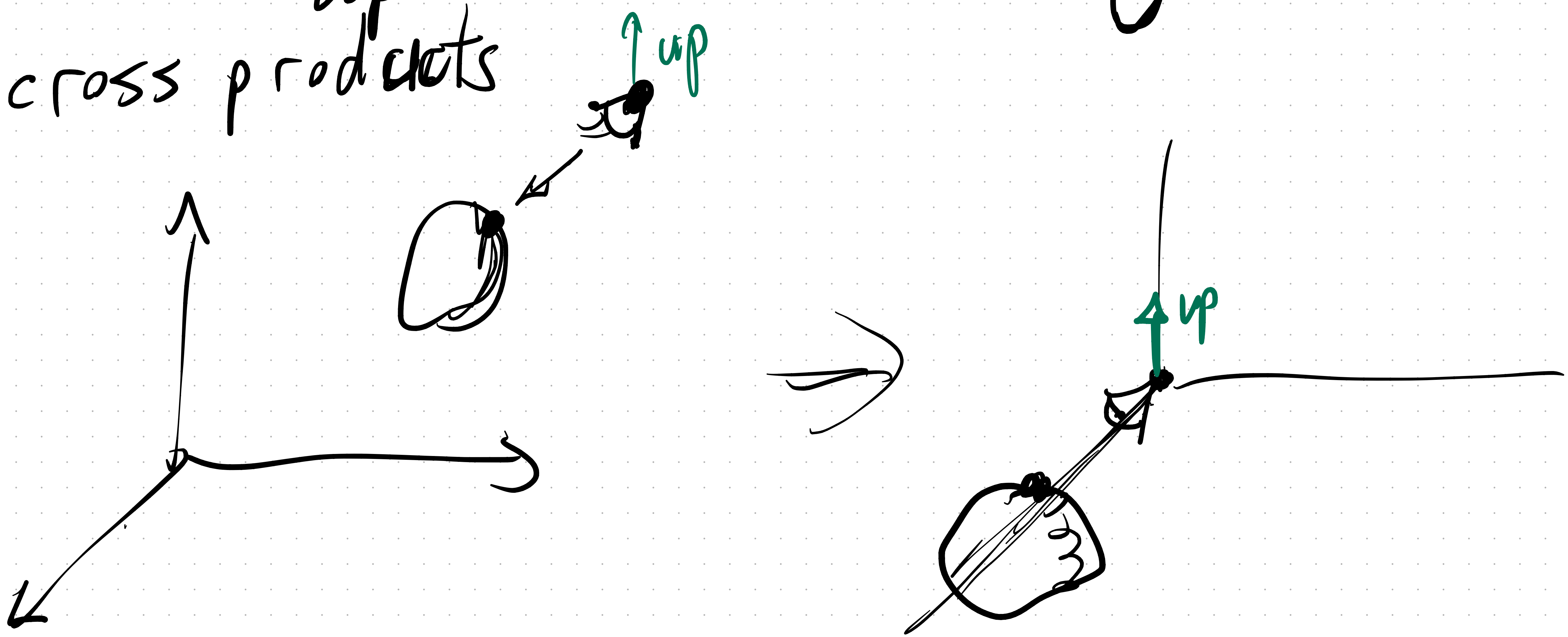
Lookat(eye, center, <u>up</u>)

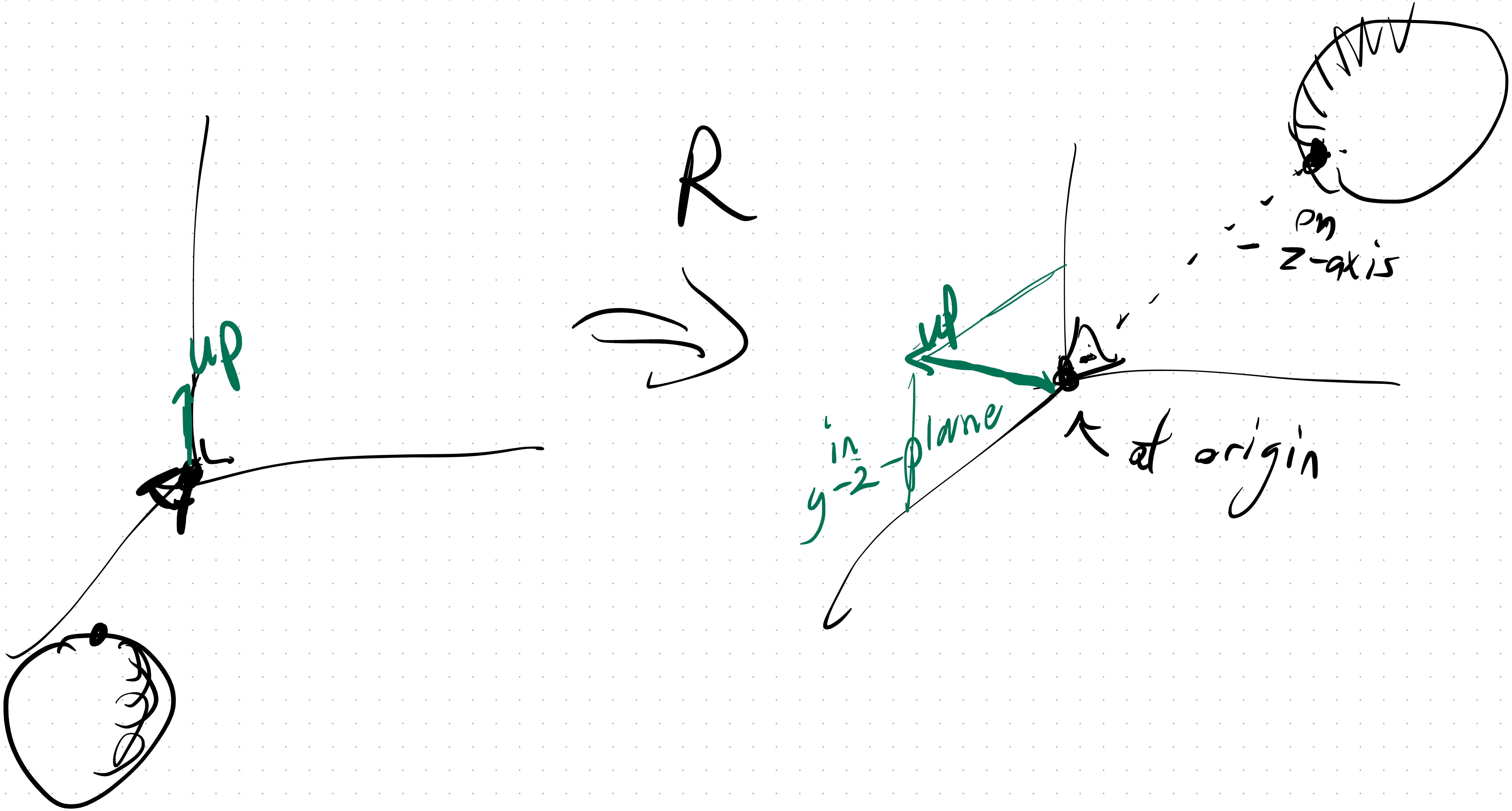
want; eye moved to (0,0,0)

center on -z axis

up vector lie in yz-plane

cross products




up
R
up
in y-z-plane
at origin
on z-axis

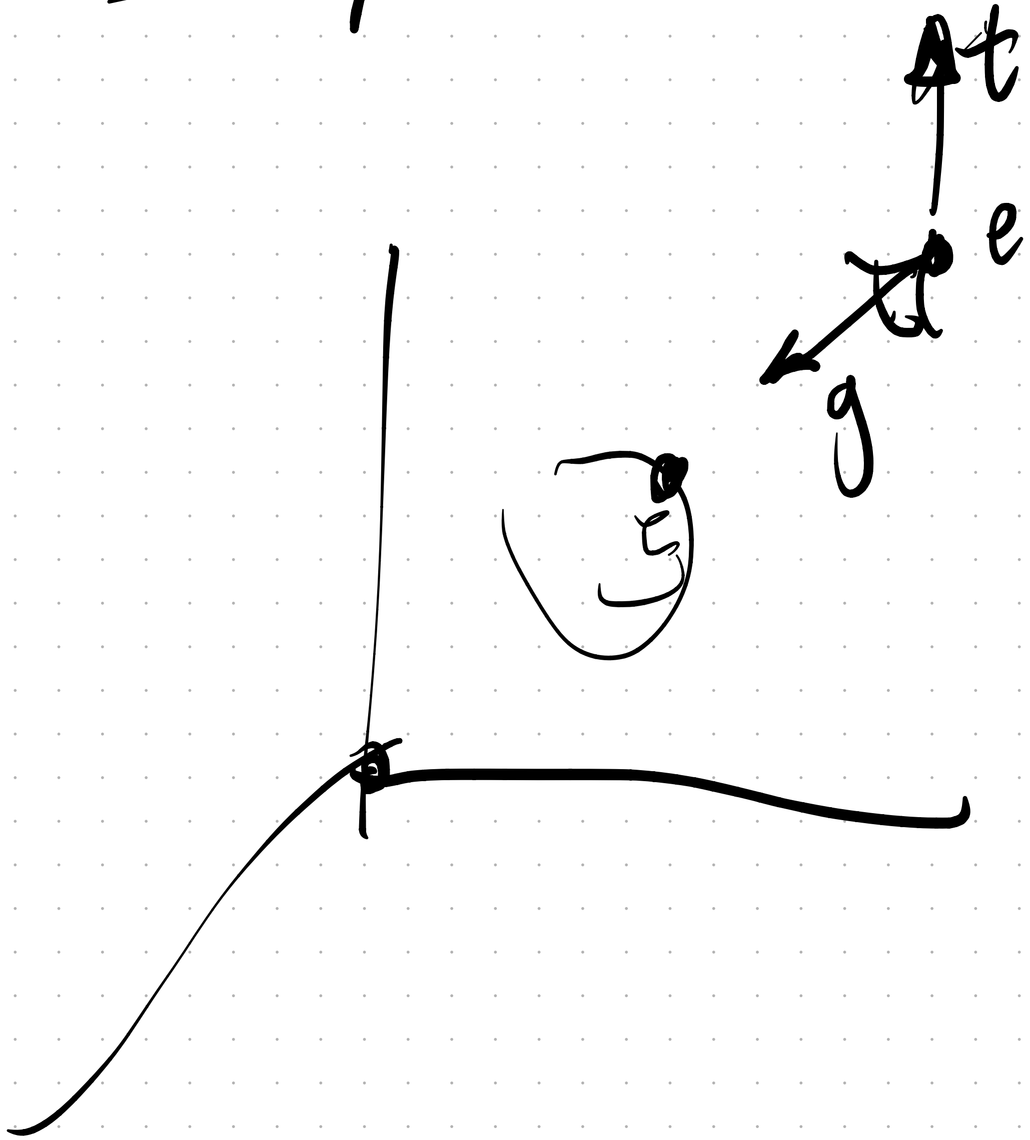
e = eye position (eye)

g = gaze direction (center - eye) ⇒ want

t = up direction (up)

u
v
w

othonormal basic vectors

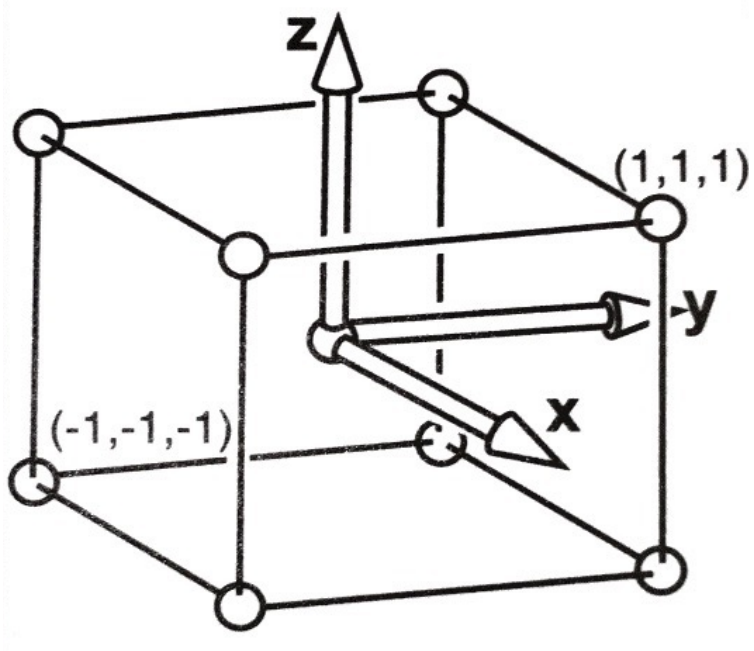


$$w = \frac{-g}{|g|}$$

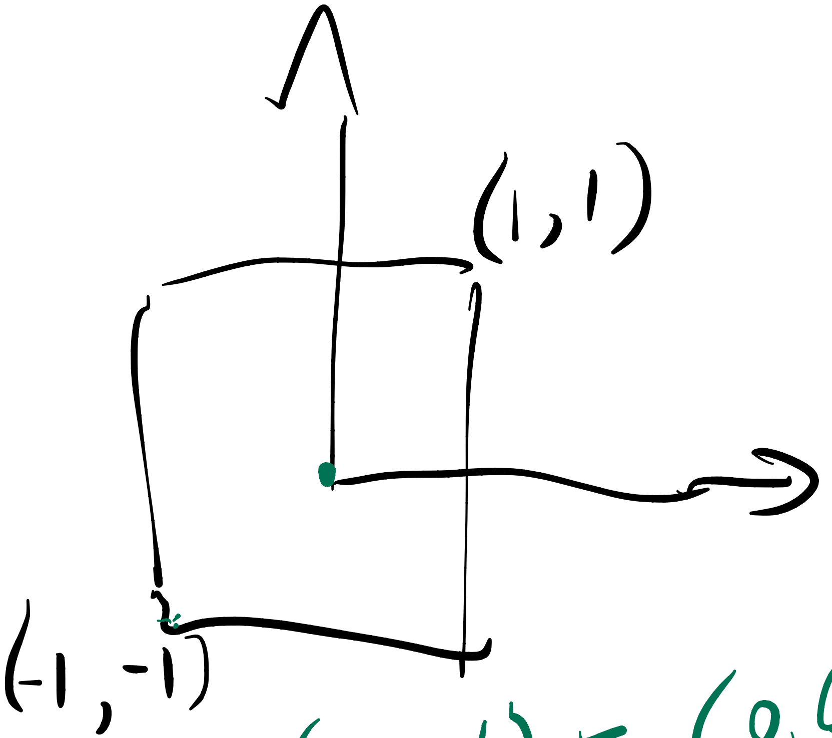
$$u = \frac{t \times w}{|t \times w|}$$

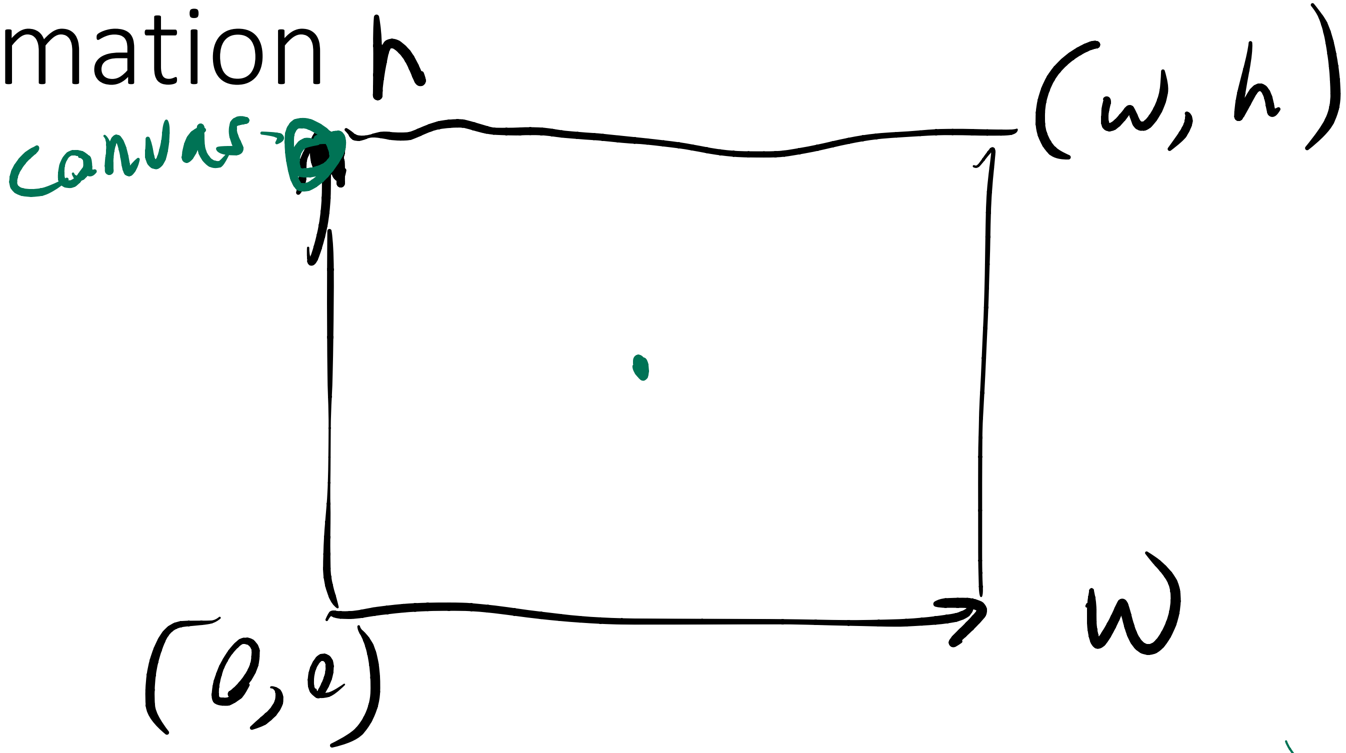
$$v = w \times u$$

Canonical View Volume



Viewport Transformation





$$(-1,-1) = (0,0)$$
$$(0,0) = \left(\frac{w}{2}, \frac{h}{2}\right)$$
$$(1,1) = (w,h)$$

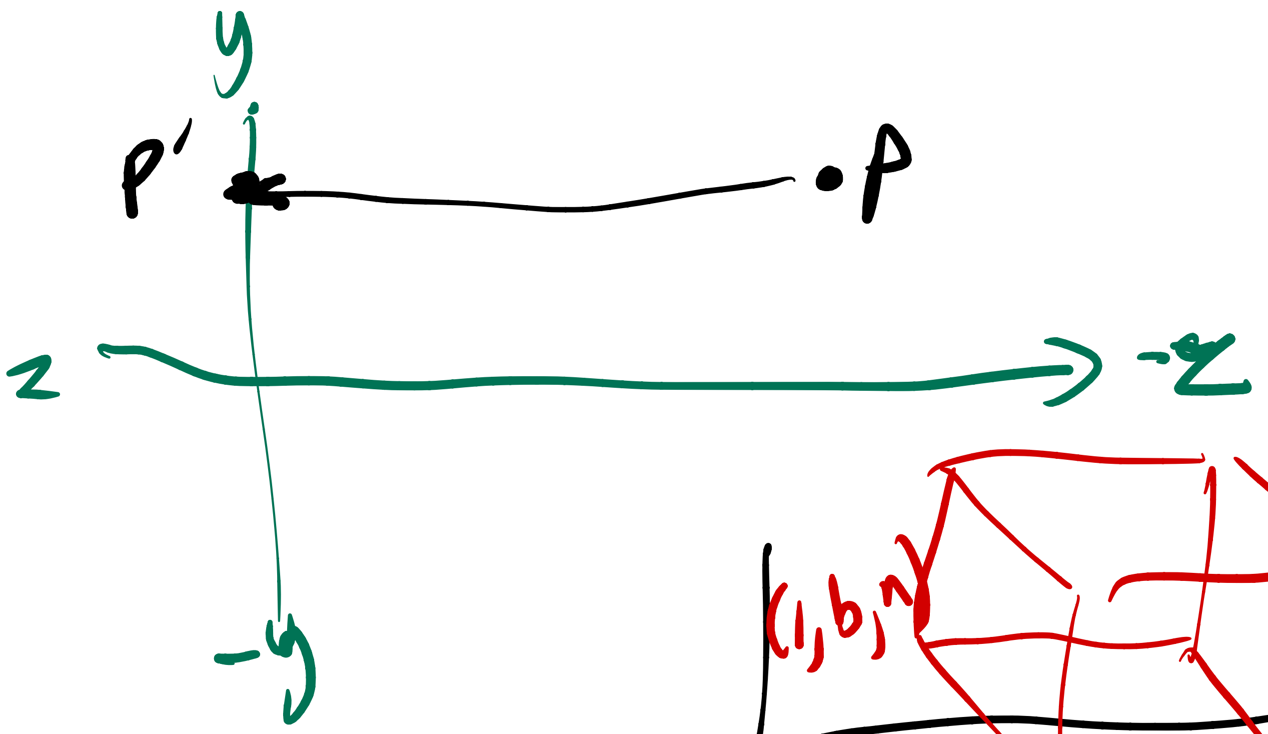
$$T\left(\frac{w}{2}, \frac{h}{2}\right) S\left(\frac{w}{2}, \frac{h}{2}\right)$$

Viewport Transformation

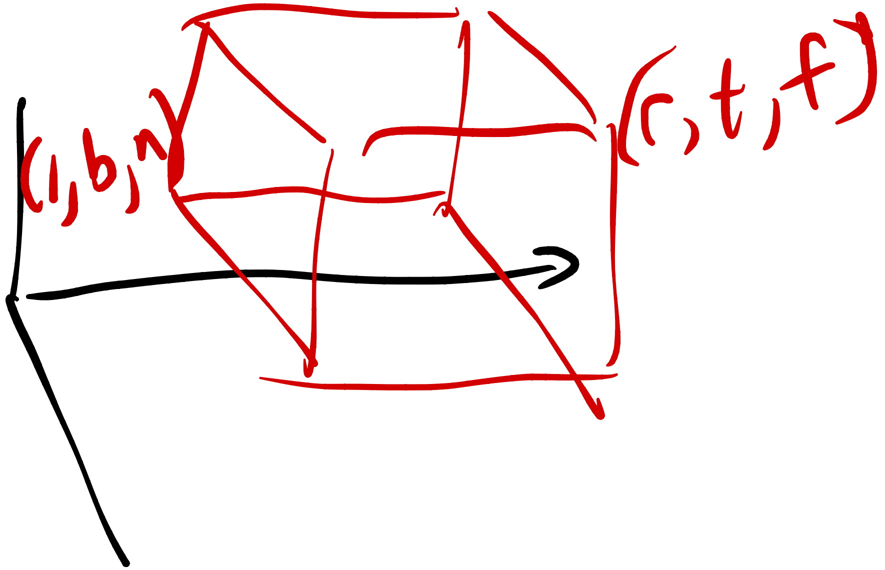
$$M_{\text{vp}} = \begin{bmatrix} \frac{n_x}{2} & 0 & 0 & \frac{n_x - 1}{2} \\ 0 & \frac{n_y}{2} & 0 & \frac{n_y - 1}{2} \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$n_x = w$
$n_y = h$

Orthographic Projection Transform



$$p = (x, y, z)$$
$$p' = (x, y, 0)$$



$x \in [\text{left}, \text{right}]$

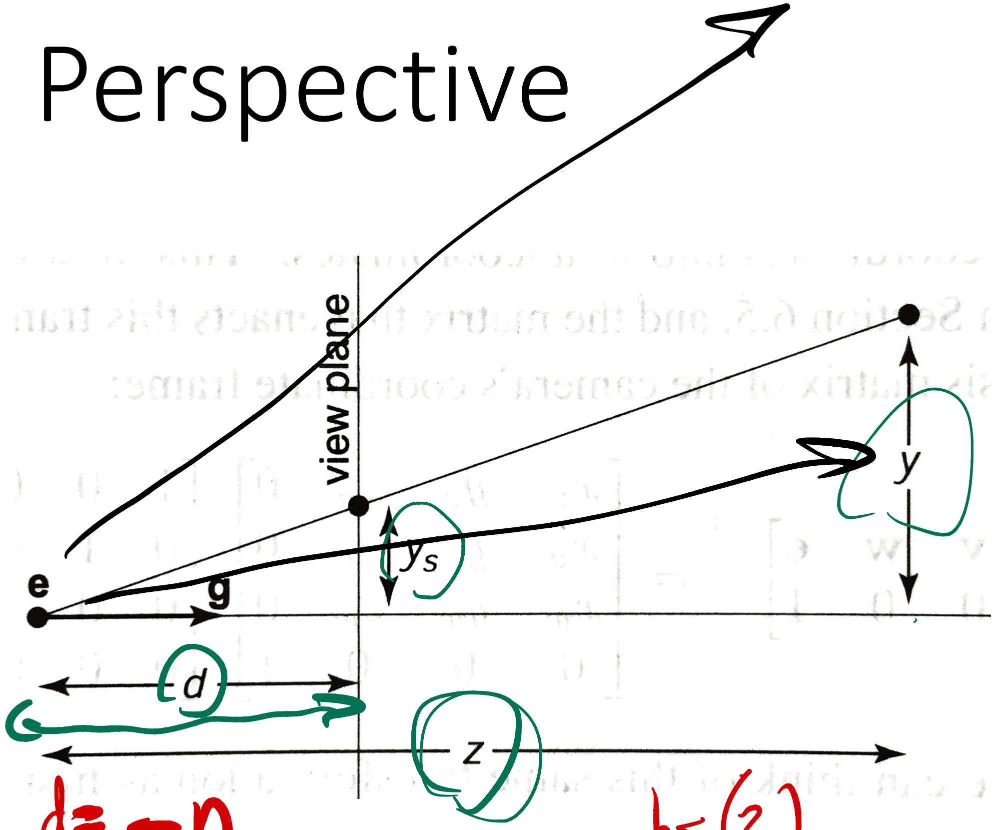
$y \in [\text{bottom}, \text{top}]$

$z \in [\text{near}, \text{far}]$

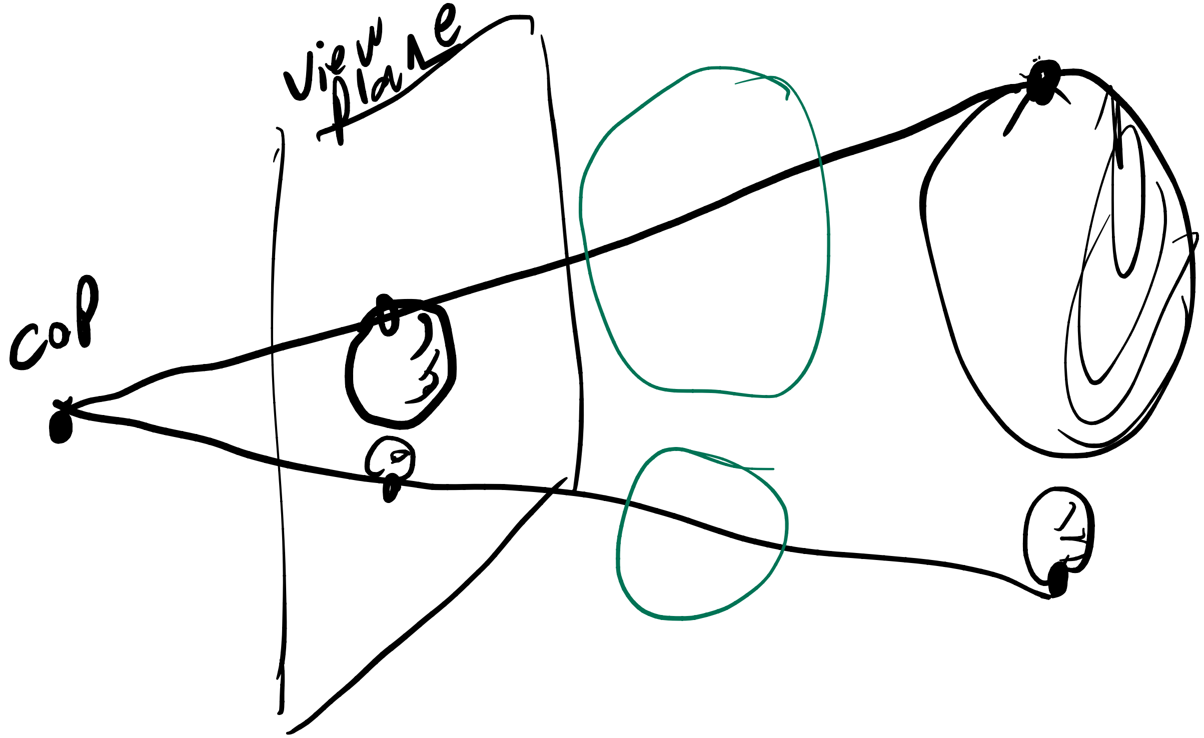
Orthographic Projection Transform

$$\mathbf{M}_{\text{orth}} = \begin{bmatrix} \frac{2}{r-l} & 0 & 0 & -\frac{r+l}{r-l} \\ 0 & \frac{2}{t-b} & 0 & -\frac{t+b}{t-b} \\ 0 & 0 & \frac{2}{n-f} & -\frac{n+f}{n-f} \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Perspective



$d = -n$ $\quad$ $z = \text{abs}(z)$



$$\frac{y_s}{d} = \frac{y}{z}$$

$$y_s = \frac{y\,d}{z} = \frac{-y\,n}{z}$$

$$V = \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

$$V' = \begin{bmatrix} x \cdot n \\ y \cdot n \\ z(n+f) - fn \\ z \end{bmatrix} = \begin{bmatrix} x \\ y \\ z \\ w \end{bmatrix}$$

$[0, 0, n, 1]$

$[0, 0, f, 1]$

$$\mathbf{P} = \begin{bmatrix} n & 0 & 0 & 0 \\ 0 & n & 0 & 0 \\ 0 & 0 & n+f & -fn \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

$$V'' = \frac{V'}{w} = \begin{bmatrix} \frac{x \cdot n}{z} \\ \frac{y \cdot n}{z} \\ n+f - \frac{fn}{z} \end{bmatrix}$$

$n =$

$n^2 + nf - fn$

$= n^2 / n$

f

$= fn + f^2 - fn$

$= f^2 / \cancel{f}$

$$(0, 0, n) \Rightarrow$$

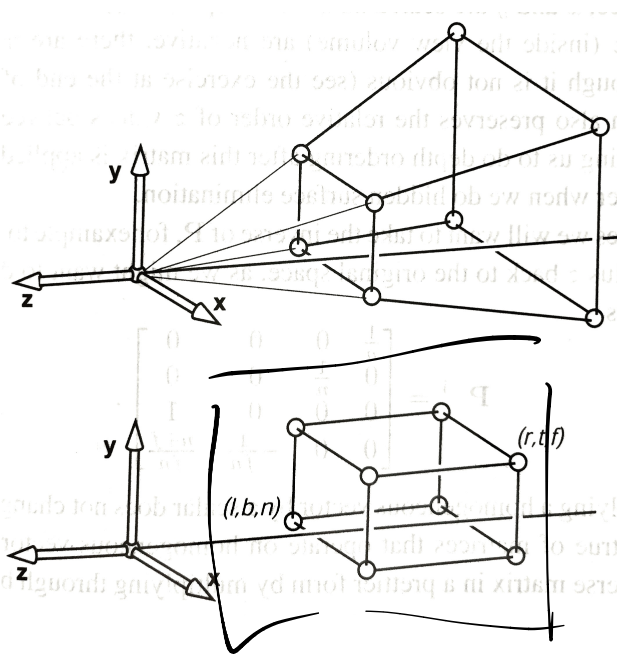
$$= \begin{bmatrix} \frac{x \cdot n}{2} \\ \frac{y \cdot n}{2} \\ n+f - \frac{fn}{2} \end{bmatrix}$$

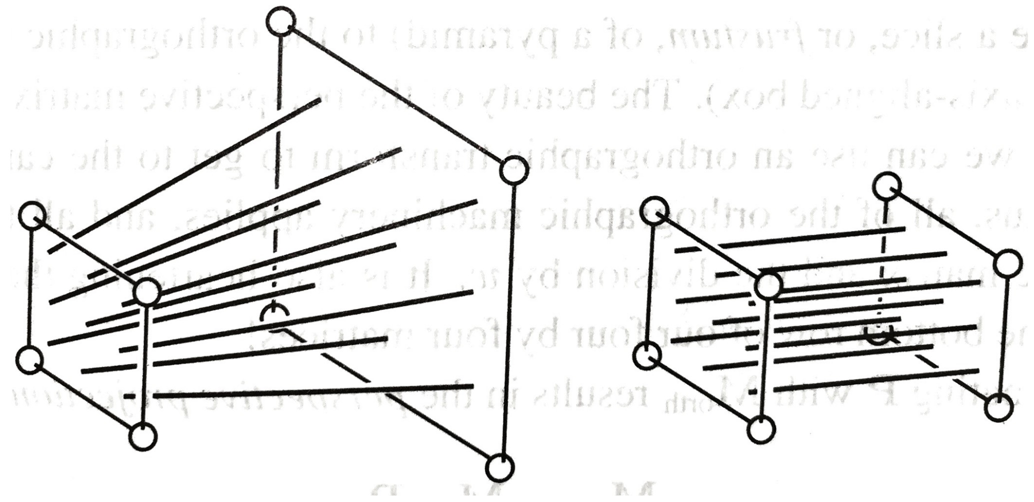
Homogeneous coordinates

$$\begin{bmatrix} x \\ y \\ z \\ w \end{bmatrix} \Rightarrow \begin{bmatrix} x' \\ y' \\ z' \\ 1 \end{bmatrix}$$

all that when you divide w gives the same are considered equal

Properties of M_{per}

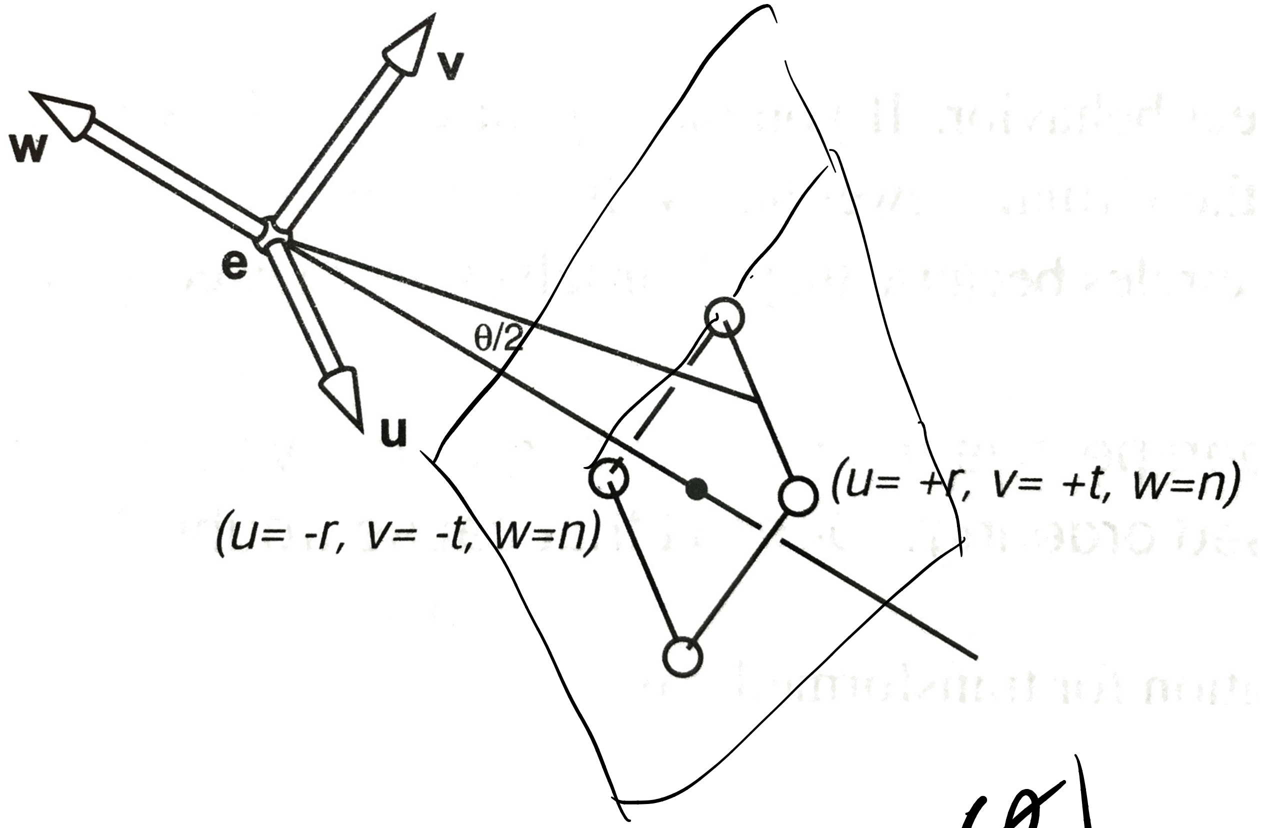


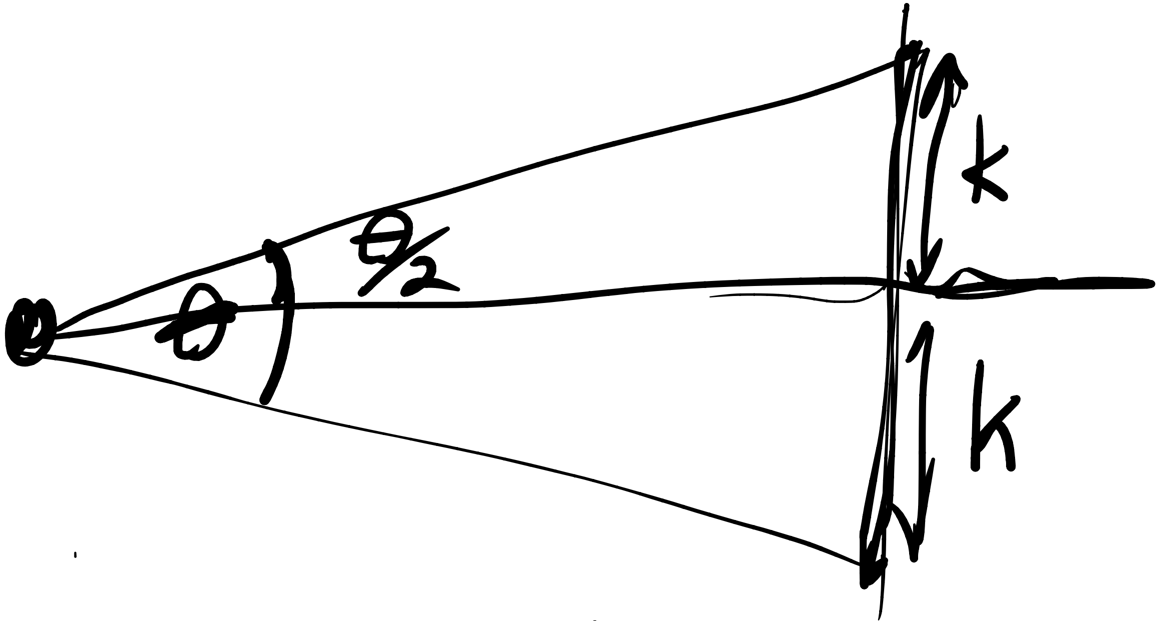


$$M_{per} = M_{orth}\ P$$

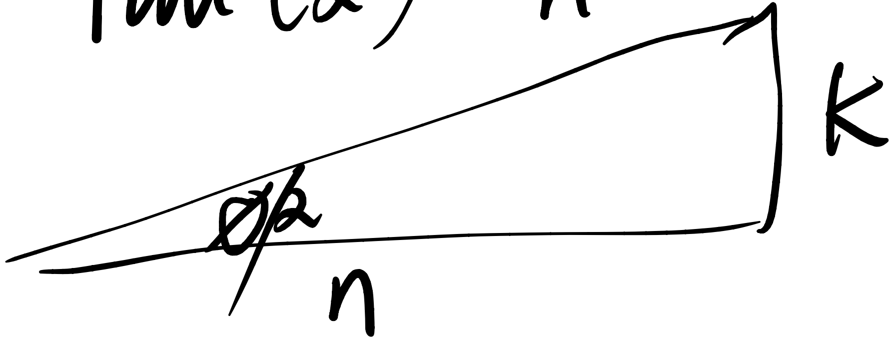
$$\mathbf{M}_{\text{per}} = \begin{bmatrix} \frac{2n}{r-l} & 0 & \frac{l+r}{l-r} & 0 \\ 0 & \frac{2n}{t-b} & \frac{b+t}{b-t} & 0 \\ 0 & 0 & \frac{f+n}{n-f} & \frac{2fn}{f-n} \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

Field of View



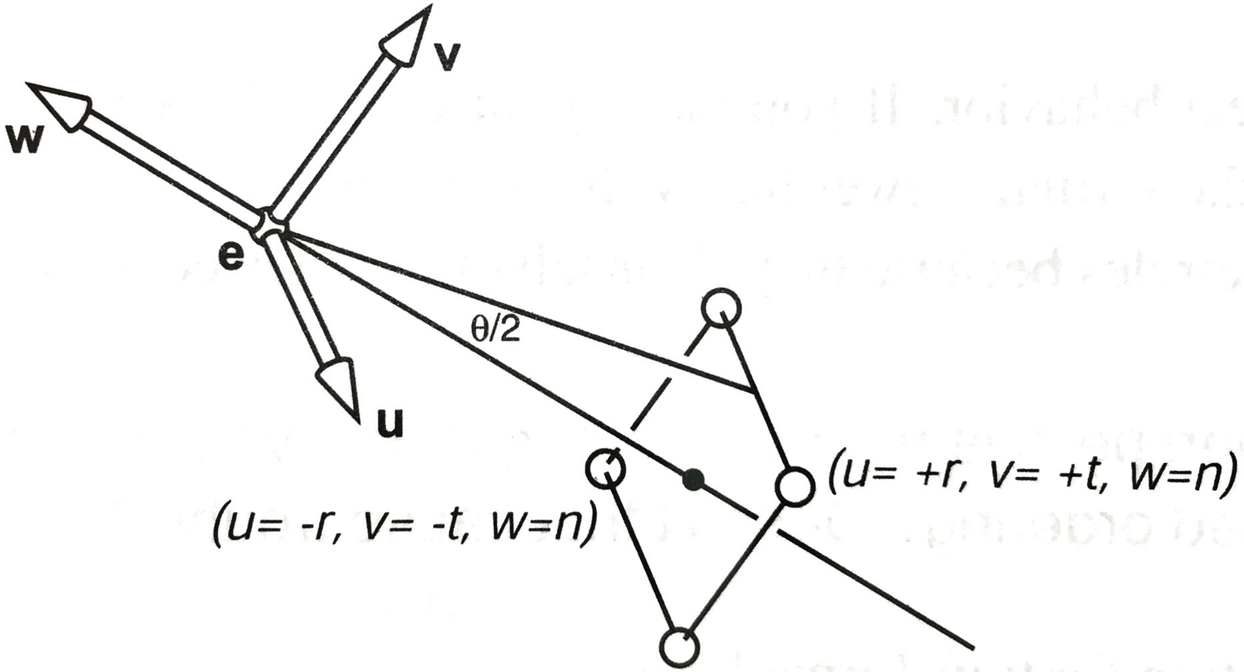


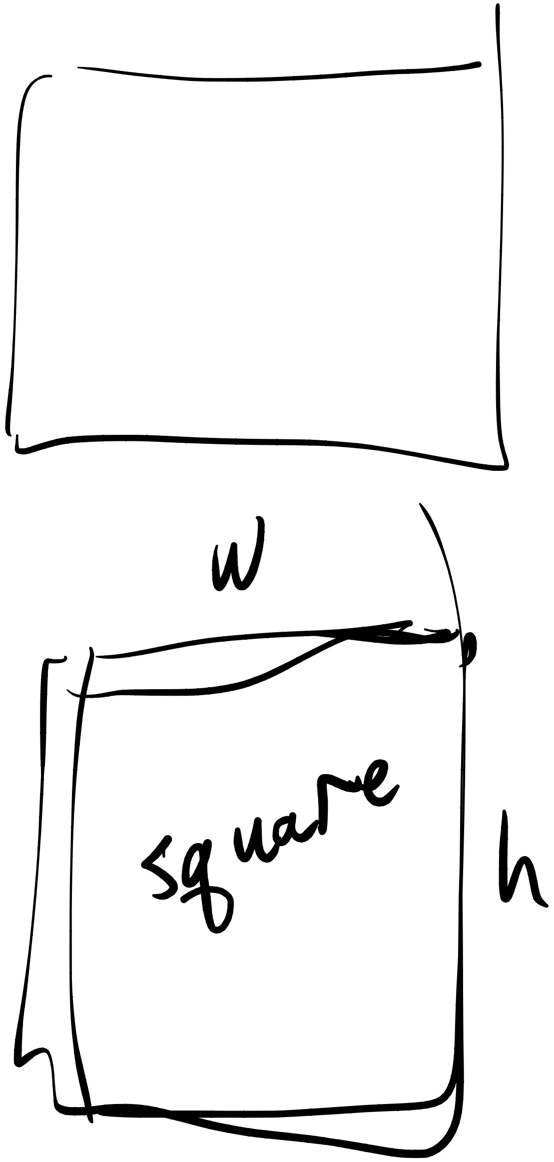
$$\tan\left(\frac{\theta}{2}\right) = \frac{k}{n}$$



$$k = \tan\left(\frac{\theta}{2}\right) \cdot n$$

Field of View



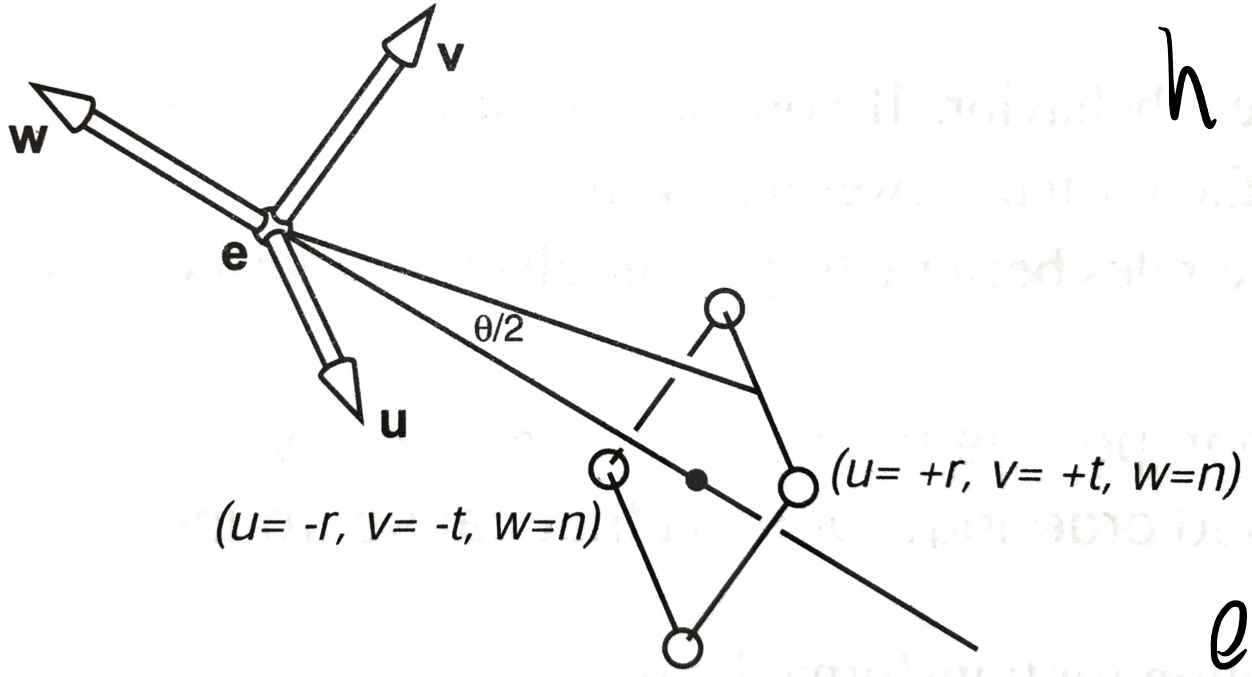


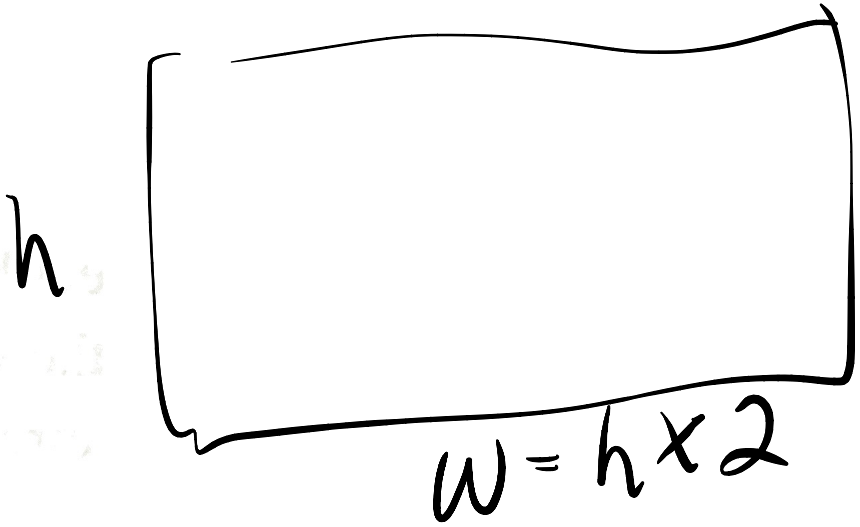
fov ⇒ in y or in x

"fovy"

w=h

Field of View





ortho = (l, r, b, t, n, f)

(-2k, 2k, -k, k, n, f)