

# 6 – lines

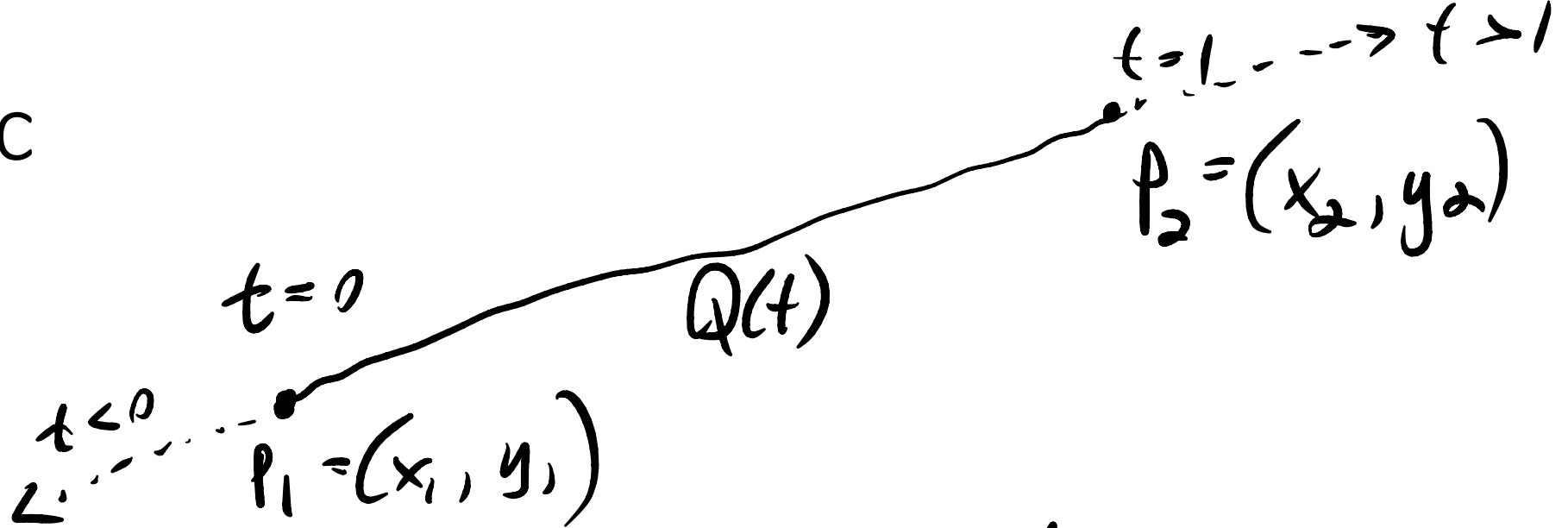
Parametric and Implicit forms of Lines

Rasterizing Lines

*And Alb if time!*

# Parametric

$$0 \leq t \leq 1$$



$$\begin{aligned}x(t) &= x_1 + t(x_2 - x_1) = x_1 + t * dx \\y(t) &= y_1 + t(y_2 - y_1) = y_1 + t * dy\end{aligned}$$

if  $|V|=1$   
 $t = \text{distance}$

$$Q(t) = P_1 + t(P_2 - P_1) = P_1 + t(V)$$

↳ vector of the dis

# Implicit Line Equation

define function  $f(x, y)$

on line  $\rightarrow f(x, y) = 0$

off line  $\rightarrow f(x, y) \neq 0$

$$y = mx + B$$

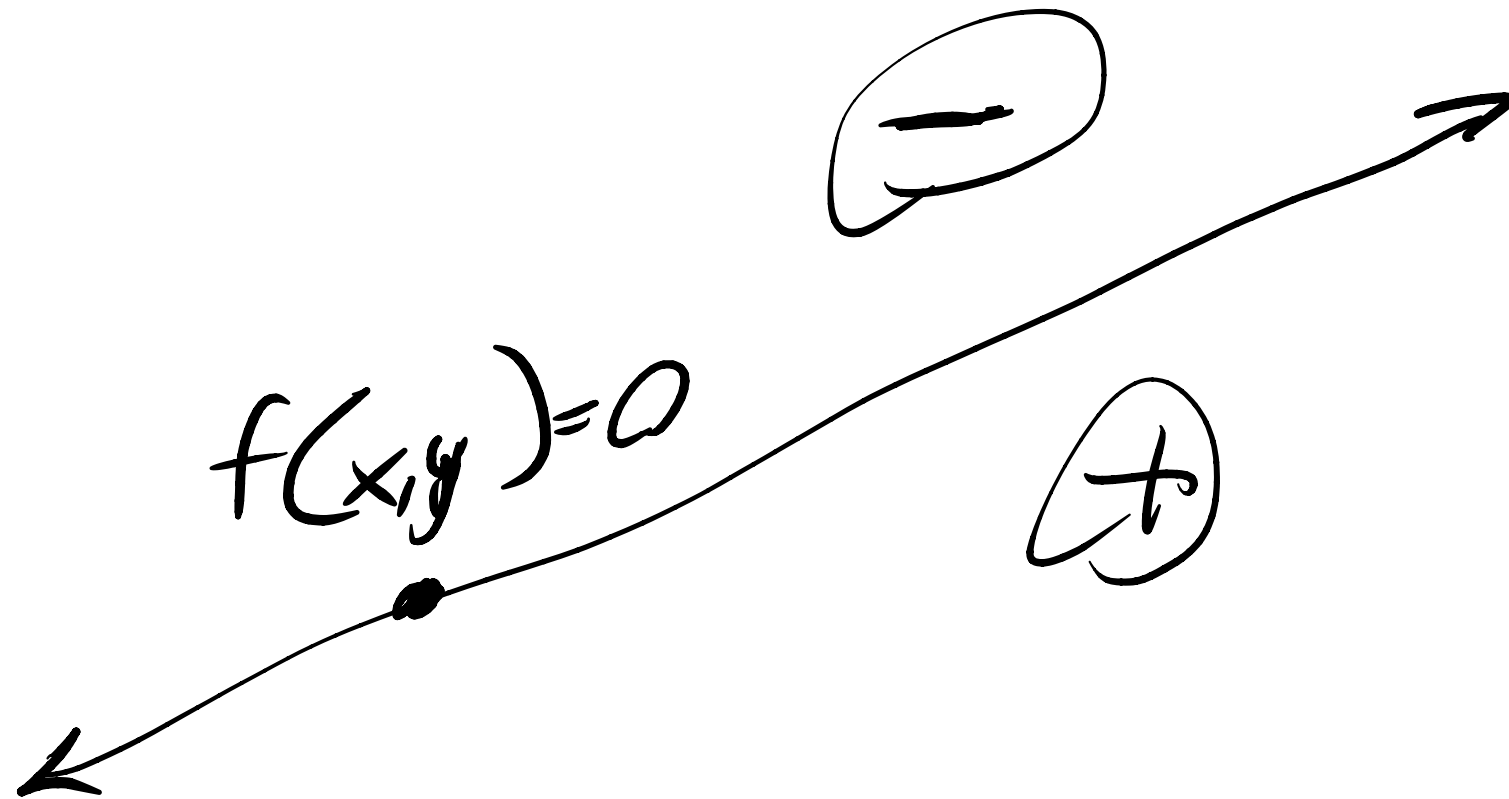
$$f(x, y) = mx + B - y$$

$$f(x, y) = ax + by + c = 0$$

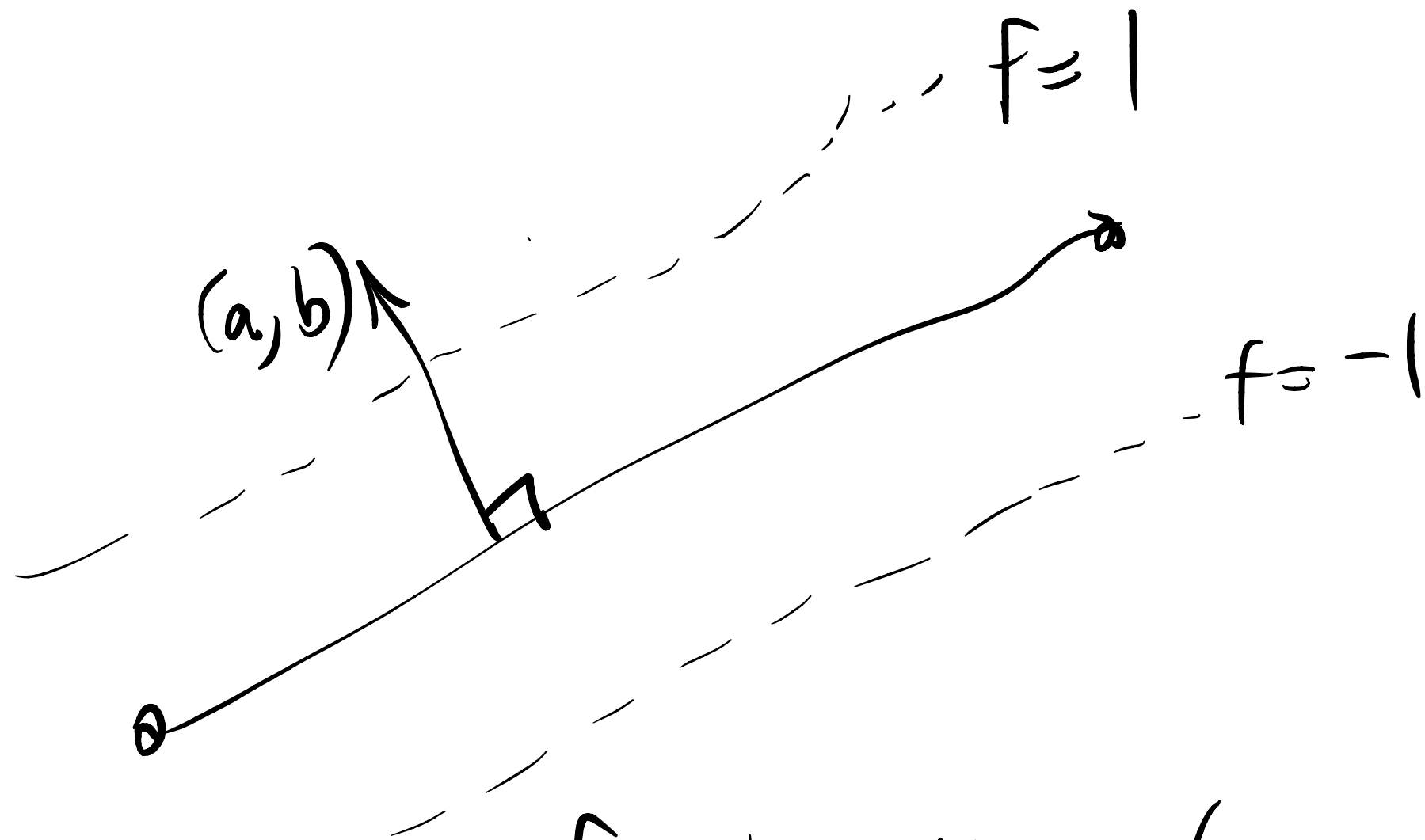
for some  $a, b, c$

$$\text{for some } \lambda \quad \lambda ax + \lambda by + \lambda c = 0$$

$a, b, c$  are coefficients of a particular line

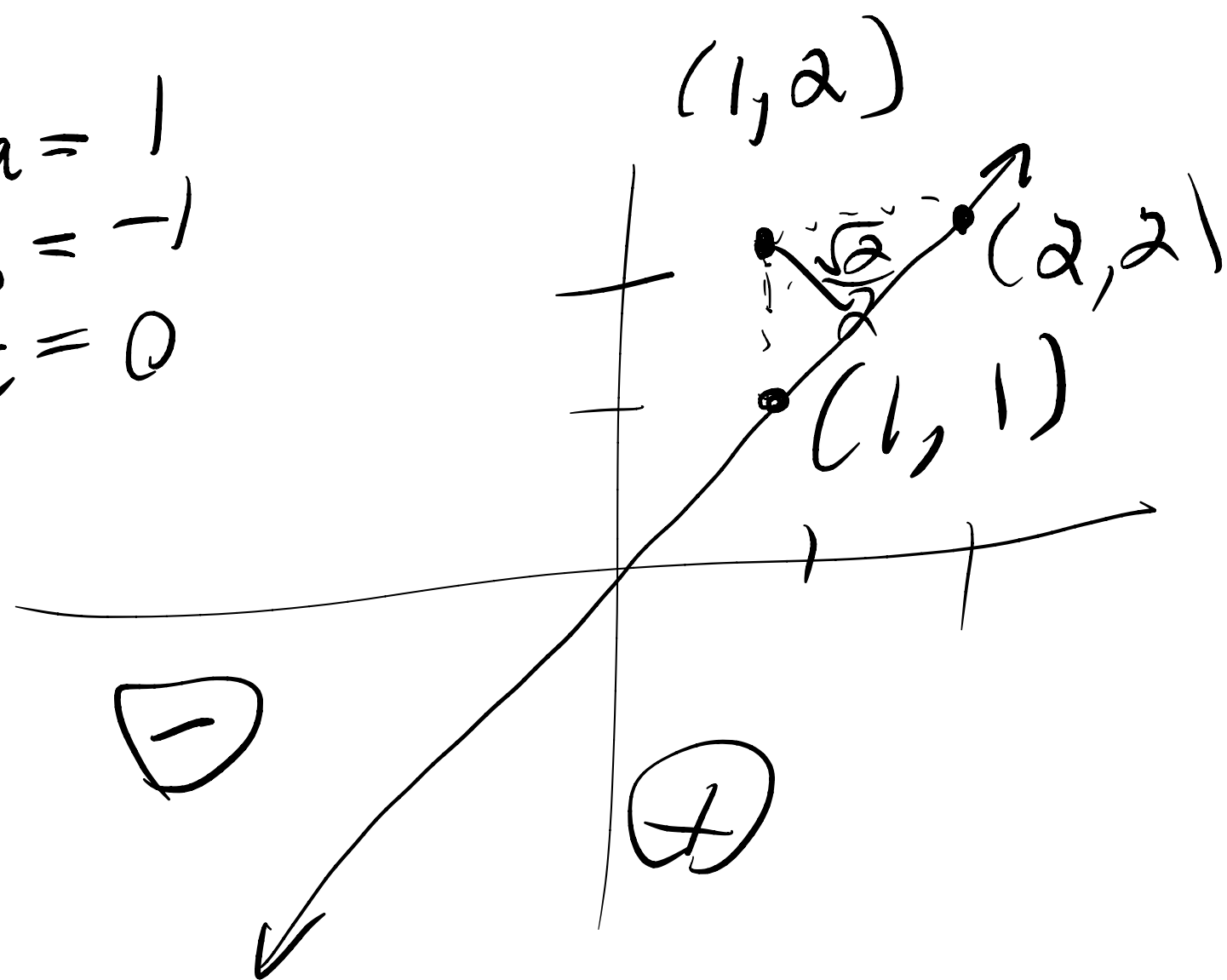


IF  $a^2 + b^2 = 1$  then  $f(x, y)$  gives the distance to line



$c$  is distance of the line to origin

$$\begin{aligned} a &= 1 \\ b &= -1 \\ c &= 0 \end{aligned}$$

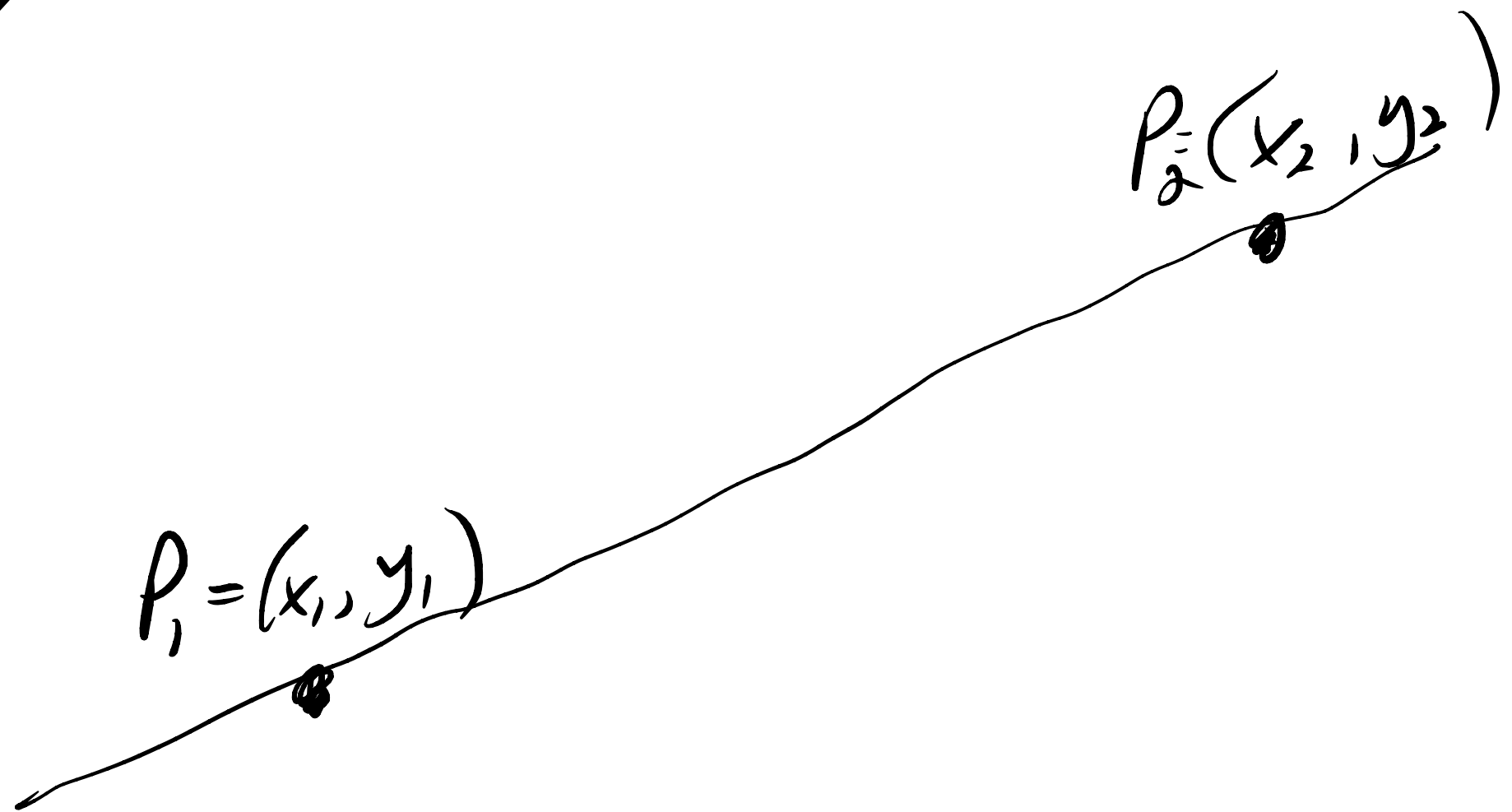


$$\begin{aligned} f(1, 2) &= 1 - 2 + 0 \\ &= -1 \end{aligned}$$

$$\frac{\sqrt{2}}{2} \quad \frac{1.41}{2} \approx 0.707$$

Math (2.5.?)  
discusses  
implicit lines

find  $a, b, c$ ?



$$f(x, y) = ax + by + c = 0$$

$$\begin{array}{r} ax_1 + by_1 + c = 0 \\ ax_2 + by_2 + c = 0 \end{array} \quad \begin{array}{l} \uparrow \\ \downarrow \end{array} \text{subtract}$$

$$a(x_1 - x_2) + b(y_1 - y_2) = 0$$

(negative  $b$ )

$$a = -b \frac{(y_1 - y_2)}{(x_1 - x_2)}$$

let  $b = 1$   
solve for  $a$   
solve for  $c$

# Rasterization

convert shapes to pixels

```
function line (x0, y0, x1, y1) {  
  let dx = x1 - x0 // assuming x1 > x0  
  let dy = y1 - y0 // ↙  
  let length = max(dx, dy)  
  let xinc = dx / length  
  let yinc = dy / length  
  let x = x0  
  let y = y0
```



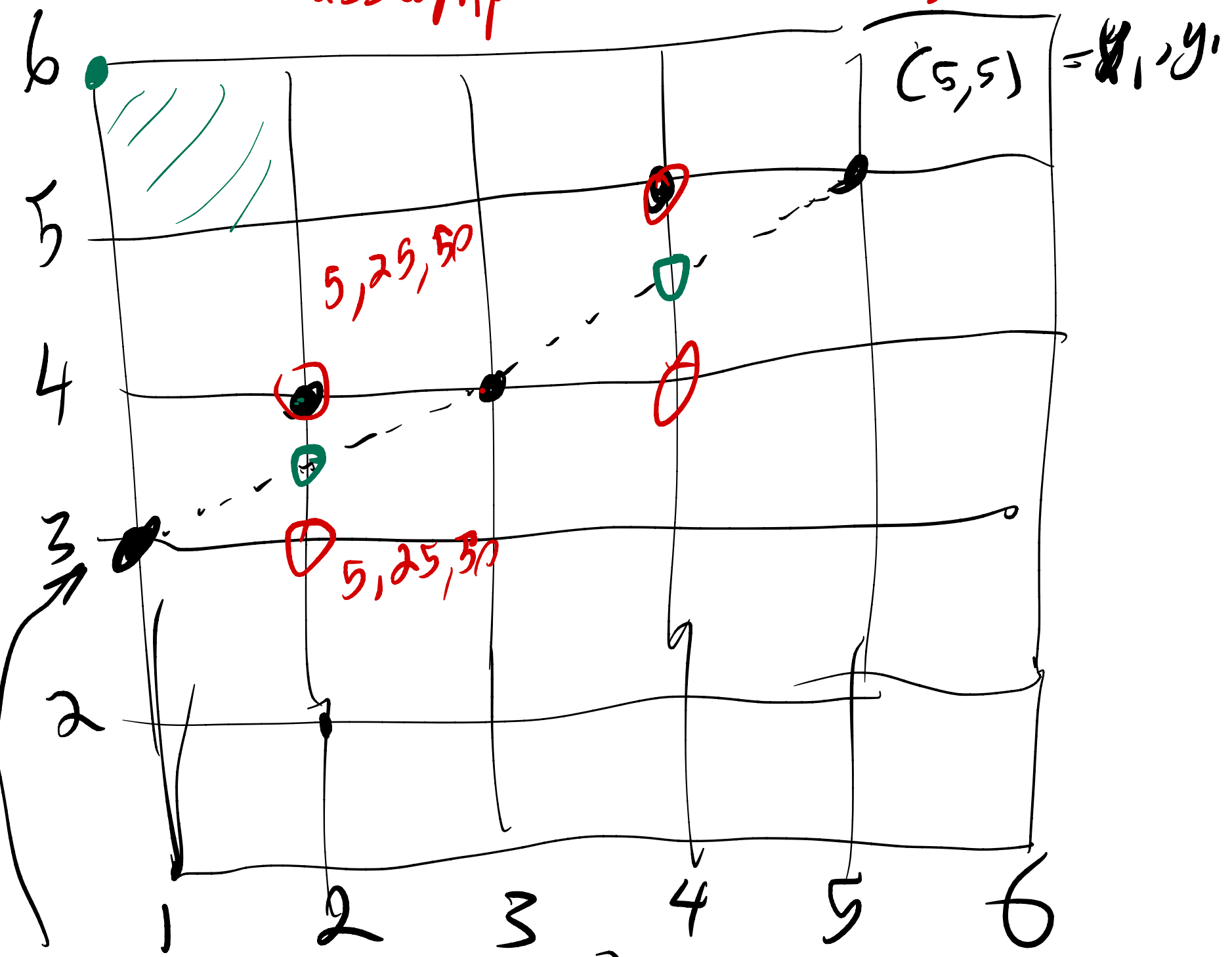
```
for (let i = 0; i < length; i++) {  
  drawPixel (round(x), round(y), 10, 50, 100)
```

```
  x += xinc
```

```
  y += yinc
```

```
}
```

assumption: black background



$$dx = 5 - 1 = 4$$
$$dy = 5 - 3 = 2$$
$$\text{length} = 4$$

$$x_{inc} = 1$$
$$y_{inc} = 0.5$$
$$x = 1, \quad y = 3$$

x	y
1	3
2	3.5
3	4
4	4.5
5	5

$(1,3) = (x_0, y_0)$   
"aliasing"

